A Toy Model of Peak Oil

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UA Low Energy Seminar
December 10, 2009

Acknowledgement

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Outline

- Hubbert’s Peak
- Logistic model of population growth
- Toy model of oil extraction
- Basis for “Decline-from-the-Midpoint”
- Basis for oil peaking before drilling peaks
- ANWR
- Conclusions

Hubbert’s Sketch of Human History: Past & Future (from *Science* in 1949)

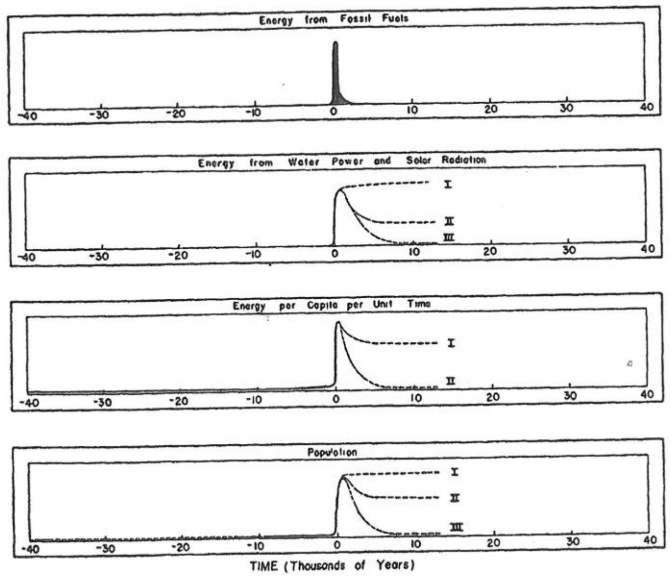


FIG. 8. Human affairs in time perspective.

Hubbert *Am. J. Phys.* 1981

“Human history can be divided into three distinct successive phases. The first, comprising all history prior to about 1800, was characterized by a small human population, a low level of energy consumption per capita, and very slow rates of change. The second, based upon the exploitation of the fossil fuels…has been a period of continuous and spectacular exponential growth. However, because of the finite resources of the Earth’s fossil fuels…the second phase can only be transitory. The third phase, therefore, must again become one of slow rates of growth, but initially at least with a large population and a high rate of energy consumption. Perhaps the foremost problem facing mankind at present is that of how to make the transition from the present exponential-growth phase to the near steady state of the future by as non-catastrophic a progression as possible.”

Population Growth Model

Growth rate of biomass equals inflow minus outflow.
Simplest case: flows are proportional to biomass.
(ingestion & immigration rates minus
excretion, metabolic, emigration, & death rates)

$$\dot{Q} = \sigma Q - \alpha Q = rQ$$

Biomass of species: Q

Constant net growth rate per unit of biomass: $\dfrac{\dot{Q}}{Q} = r$

Exponential Growth

Solution to simple growth model: $$Q = Q_0 e^{rt}$$

Exponential growth is not sustainable.

Model needs to be adjusted.

Introduce a "crowding" term: $-\gamma Q^2$

Logistic Differential Equation

$$\dot{Q} = rQ - \gamma Q^2$$

$$\dot{Q} = r\left(1 - \frac{Q}{Q_{tot}}\right)Q$$

Carrying Capacity: stable steady state to which biomass asymptotically grows.

$$Q_{tot} = r / \gamma$$

Net growth rate per unit of biomass decreases linearly as a function of biomass.

$$\frac{\dot{Q}}{Q} = r\left(1 - \frac{Q}{Q_{tot}}\right)$$

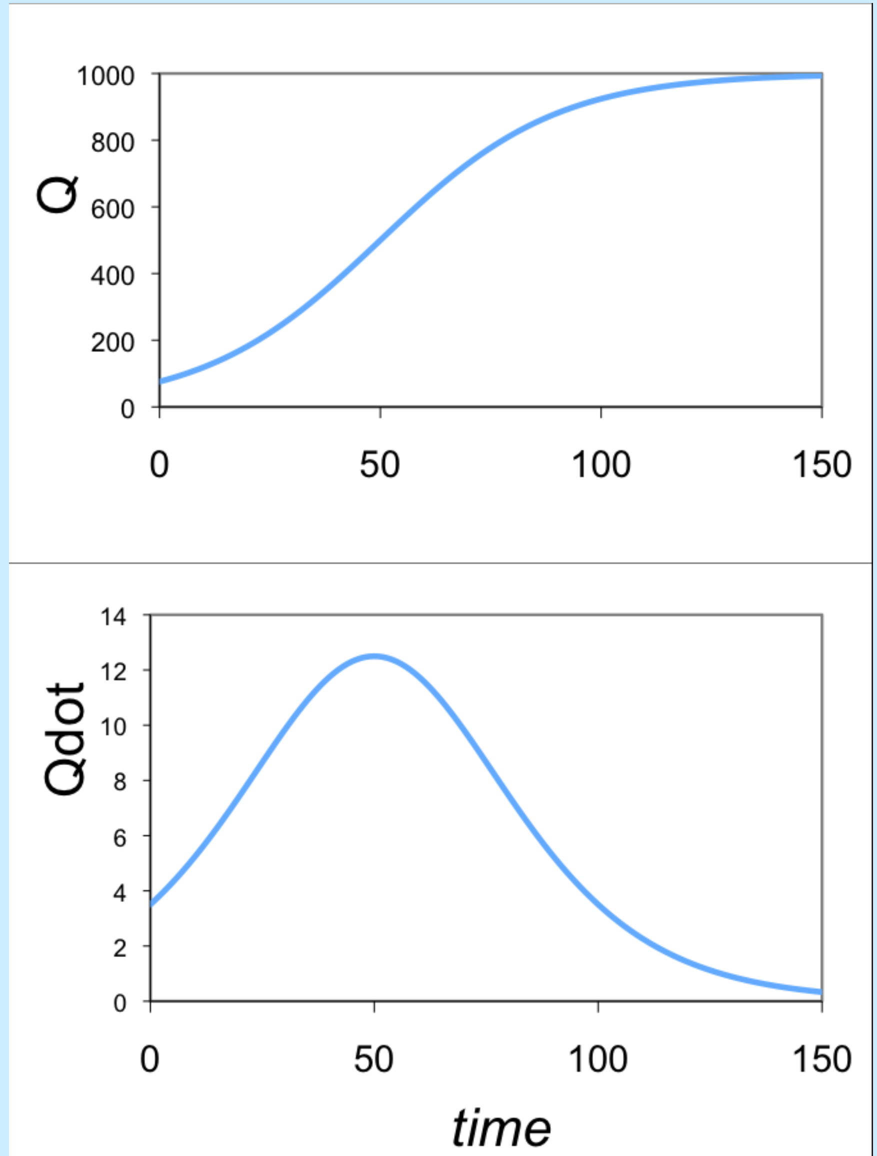
Logistic Growth Curve

$$Q = \frac{Q_{tot}}{1 + e^{r(t_m - t)}}$$

Peak at midpoint time t_m occurs when

$Q = Q_{tot}/2$

$$\dot{Q} = \frac{rQ_{tot}e^{r(t_m - t)}}{(1 + e^{r(t_m - t)})^2}$$



Logistic Growth Applied to Oil Production

In 1956 Hubbert applied a logistic growth model to oil production data:

$$\frac{P}{Q} = r\left(1 - \frac{Q}{Q_{tot}}\right)$$

Q cumulative oil production (in billions of barrels)

$P = \dot{Q}$ oil production (in billions of barrels per year)

r initial production per cumulative production (in inverse years)

Q_{tot} total recoverable oil that ultimately will be produced (in billions of barrels)

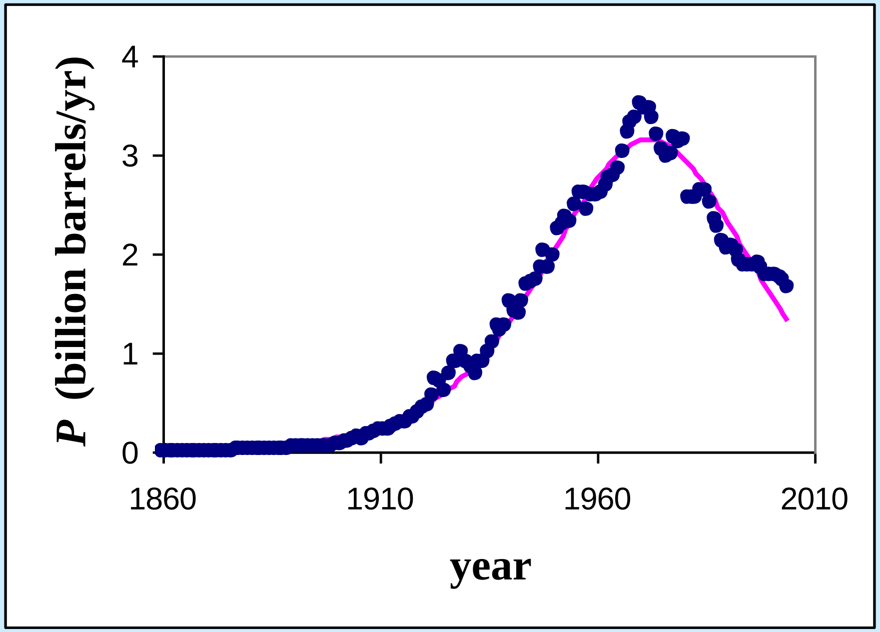
Is Dependence on Foreign Oil Inevitable?

Hubbert in 1956 correctly predicted US production would peak in 1970.

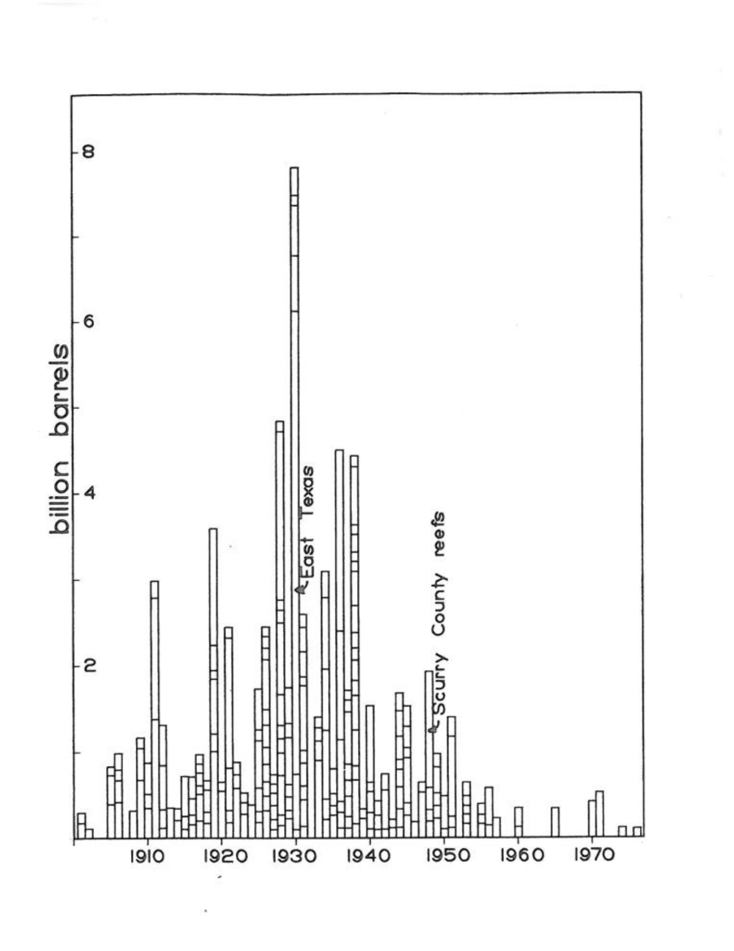
Hubbert used the derivative of the logistic growth curve to fit oil production data.

"Decline-from-the-Midpoint"

is observed in US production



Yearly oil discovery in the lower 48



Deffeyes *Hubbert's Peak* Princeton Press 2001

Typical (controversial) Prediction

“Many credible analysts have recently become much more pessimistic about the possibility of finding the huge new reserves needed to meet growing world demand.

Even the most optimistic forecasts suggest that world oil peaking will occur in less than 25 years.

The peaking of world oil production could create enormous economic disruption, as only glimpsed during the 1973 oil embargo and the 1979 Iranian oil cut-off.”

Peaking of World Oil Production, Impacts, Mitigation, and Risk Management (2005 consultant report to DOE)

Why does the logistic growth curve fit oil production data?

Logistic growth is a model for any growth process in which the per capita growth rate decreases linearly as the quantity grows.

Is there a physical explanation for bell-shaped oil production & “decline-from-the-midpoint”?

Deffeyes' "Explanation"

"…the analogy between population growth and oil production seems a little odd…The peculiar part is the analogy between people having babies and oil wells begetting additional oil wells. In a crude sense, oil wells do raise families. Drilling a discovery well brings on a bunch of new wells to develop the oil field."—Deffeyes *Hubbert's Peak* Princeton Press 2001

The Physical Process of Oil Extraction

Oil extraction is a pressure driven process.

High pressure water or gas forces oil through well.

Oil/water contact line rises as oil is depleted and pressure drops until oil is no longer forced out.

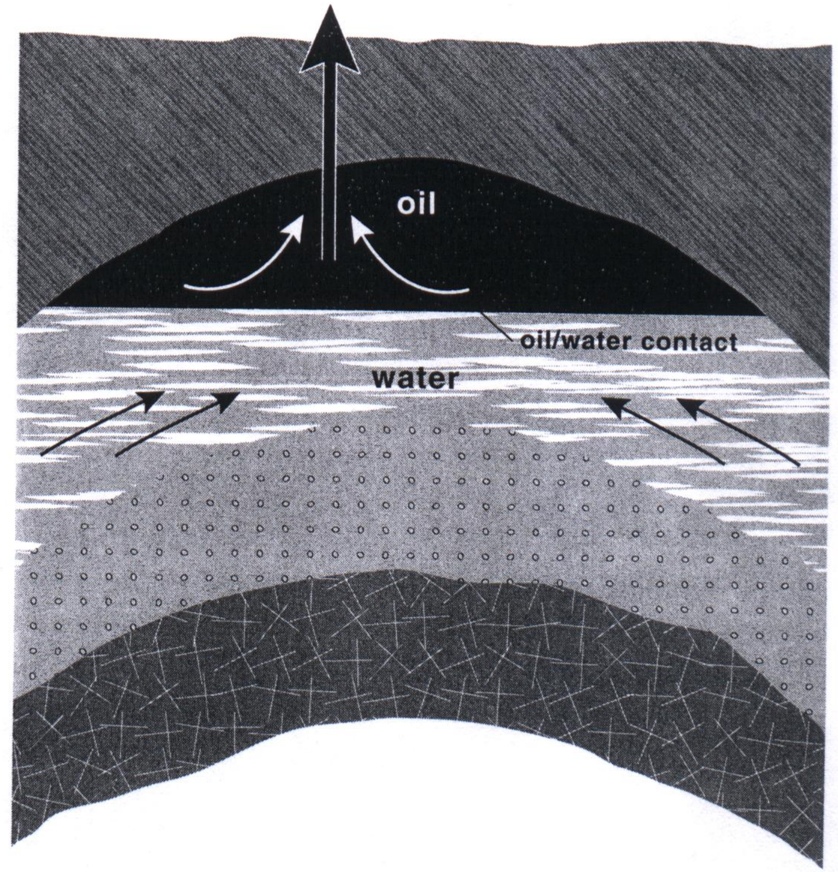


Figure 5.1 **Basic Oil Reservoir Dynamics**
SOURCE: Simmons & Company International

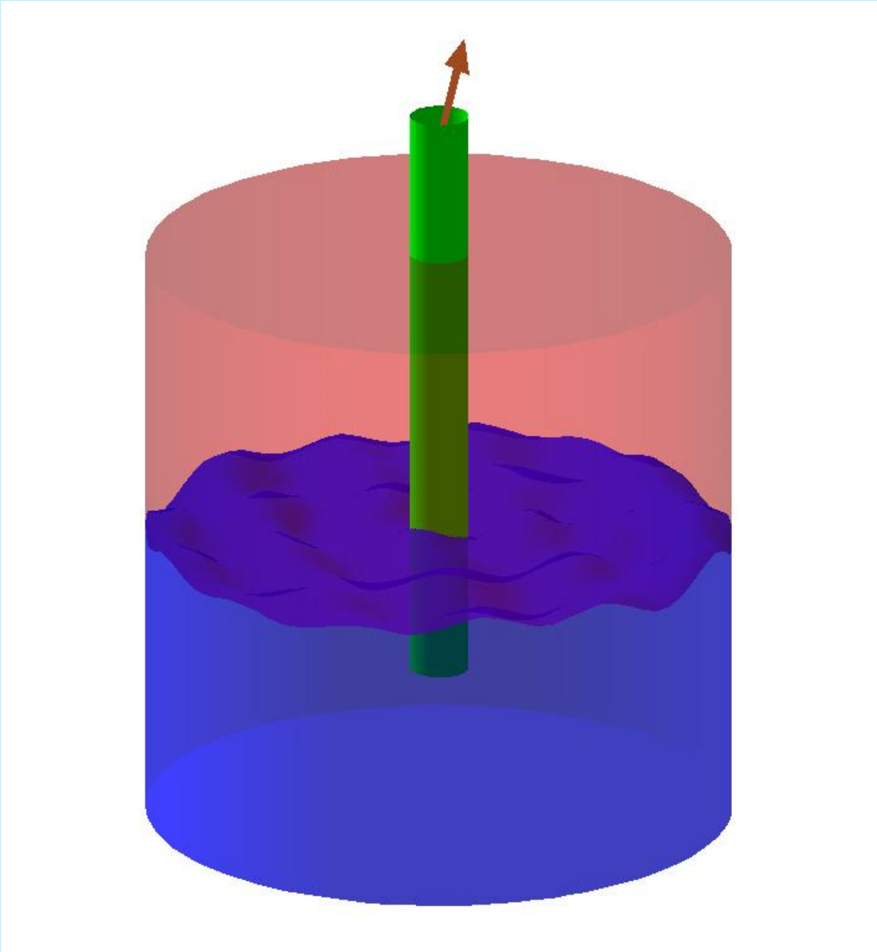
Lakeview Gusher 1910
Maximum of 90K barrels/day

The Toy Model

A sealed vessel filled with high pressure gas & oil with impurities.

Pressure-driven oil is emptied from the vessel through a tube. Gas expands, oil depletes, pressure drops.

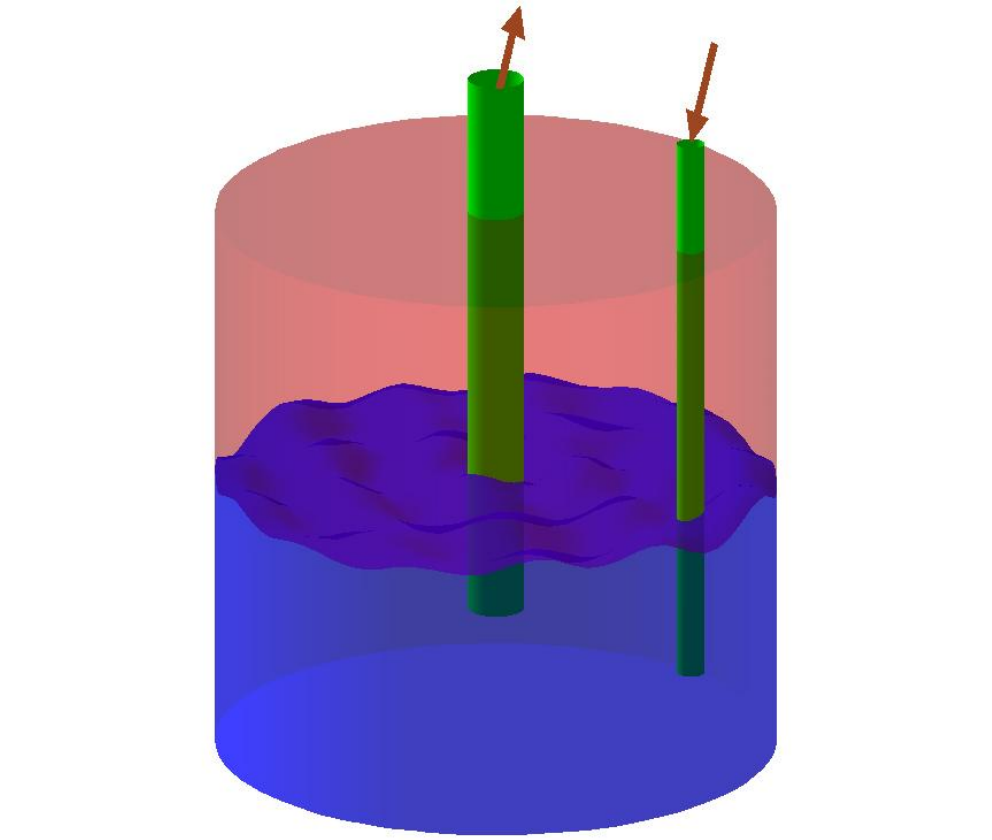
Over time, the area of tube is increased or decreased to represent more or less drilling.



Re-injection Toy Model

In real oil reservoirs, water or gas is re-injected as a secondary recovery method to maintain high pressure.

The toy model can be modified to include re-injection. Volume of oil and impurities is kept constant, but fraction of oil to impurities decreases.



The Model Equations

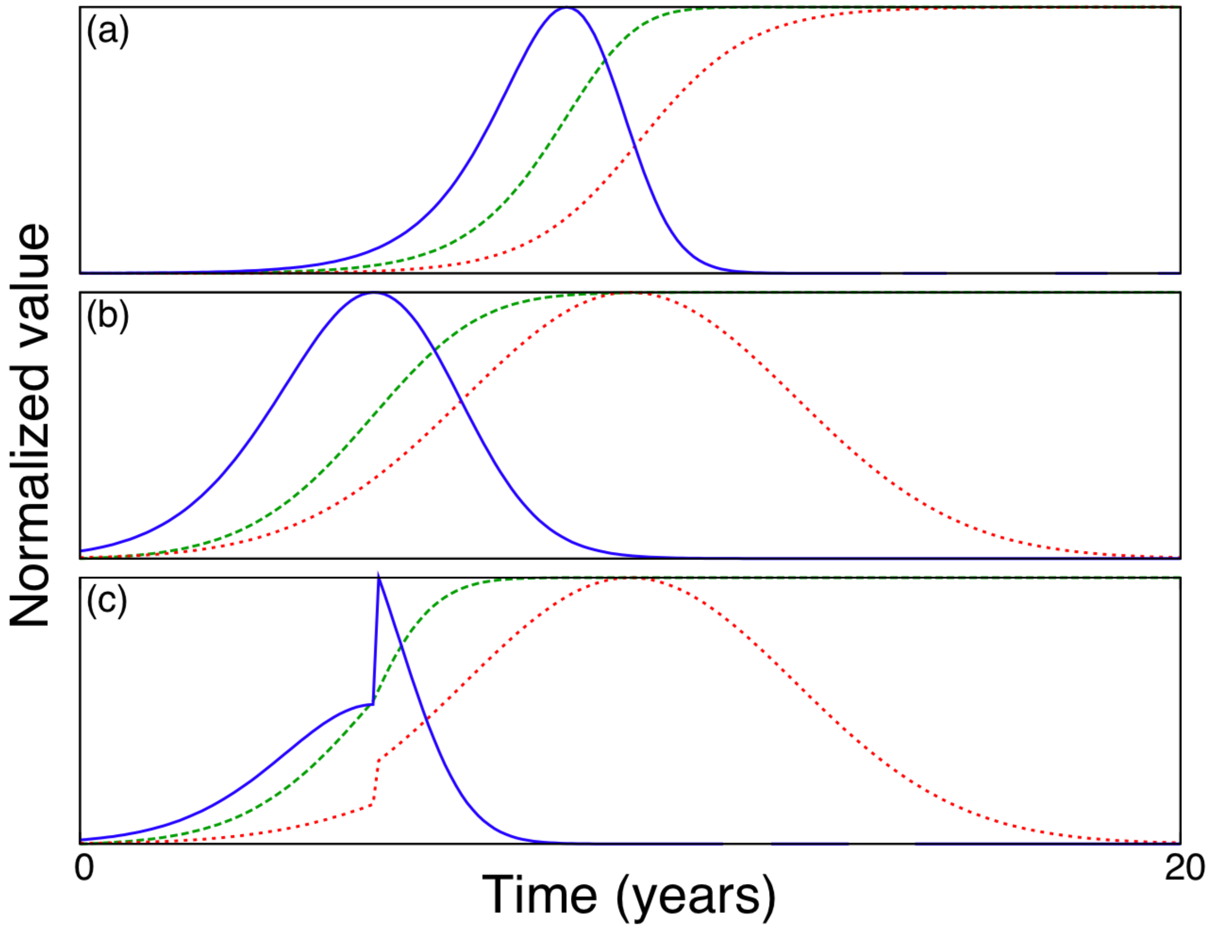
$$\dot{Q} = uAx$$

$$x = x_0(1 - Q/Q_{tot})$$

$$u = \sqrt{\frac{2(P_{wh} - P_{atm})}{\rho}} \approx \sqrt{\frac{2P_{wh}}{\rho}}$$

$$\dot{Q} = \sqrt{\frac{2P_{wh}}{\rho}} A x_0 (1 - Q/Q_{tot})$$

Behavior of model with several functions for A



Physical explanation for Peak Oil

In order to increase production, the area of wells must be increased initially. However, production must eventually decrease to zero, regardless of how the area is varied. For choices of area that mimic drilling at real oilfields, production is bell shaped and declines approximately from the midpoint.

Production peaks before drilling peaks

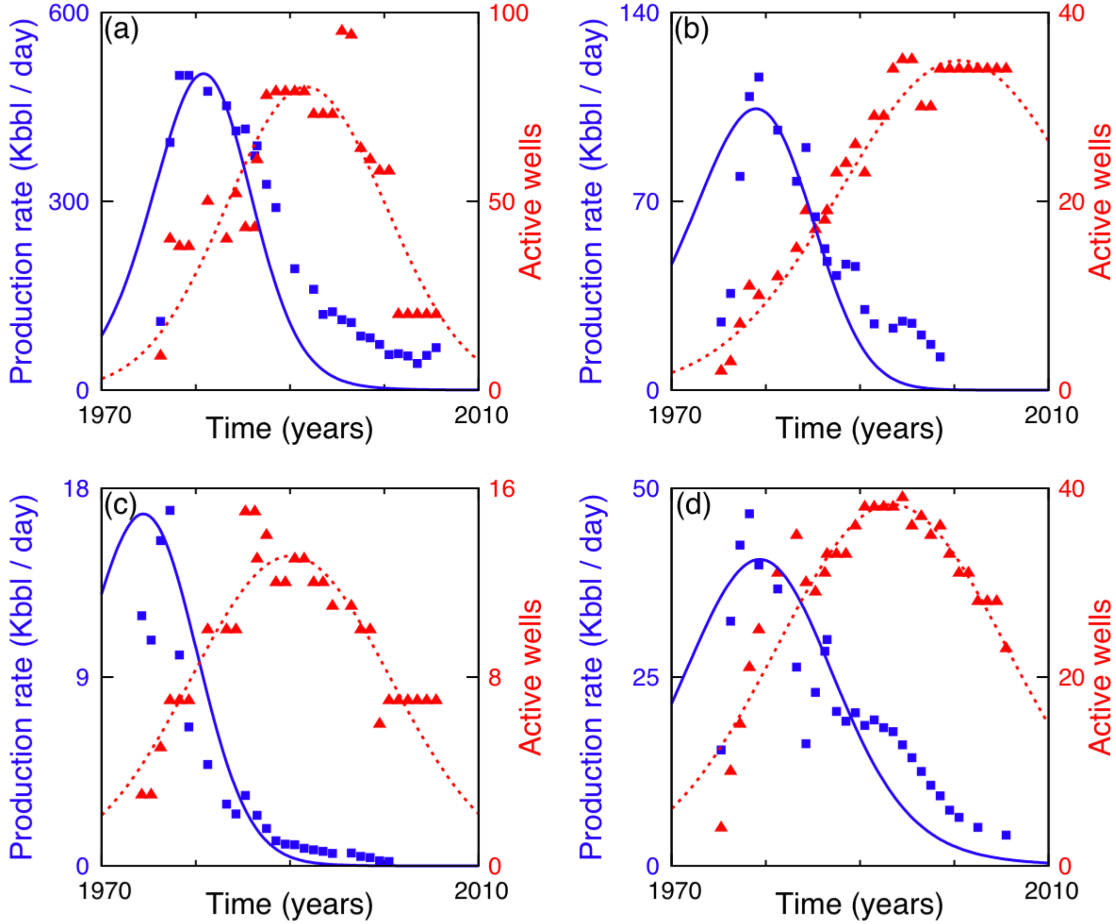
$$\dot{Q} = \sqrt{\frac{2P_{wh}}{\rho}} A x_0 (1 - Q/Q_{tot}) = Af$$

$$\ddot{Q} = \dot{A} f + A\dot{Q}\, df/dQ = 0$$

$$\dot{A} = -A\dot{Q} f^{-1} df/dQ \Rightarrow \dot{A} \geq 0$$

Area & production are always positive; f is always positive; df/dQ is always negative (since the oil fraction is monotonically decreasing). At Hubbert's peak the derivative of area must still be positive, so drilling must still be increasing when production peaks. Increased drilling can't serve as an indicator of future increased production.

Real data from U.K., Egypt, Indonesia, and Trinidad & Tobago



Drilling for Oil in the Arctic National Wildlife Refuge

To drill or not to drill?

Gas prices are rising: we need the oil to reduce dependency on foreign oil.

ANWR is a pristine ecological preserve that will be damaged irreparably by drilling.

Hubbert Peak modeling can help examine the validity of the argument in favor.

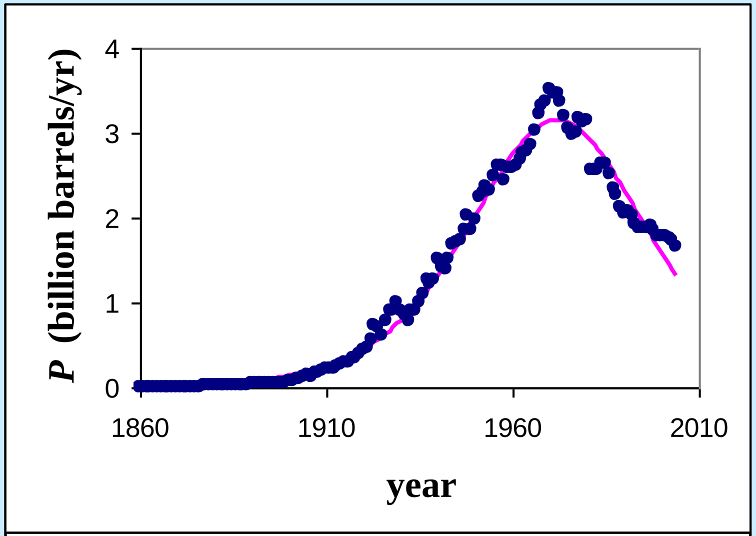
Data on U.S. Oil Production

Oil production for U.S. lower 48 states. Fit with logistic production curve:

r = 0.063 yr $^{-1}$

Q_{tot} = 201 billion bbl

t_m = 1972

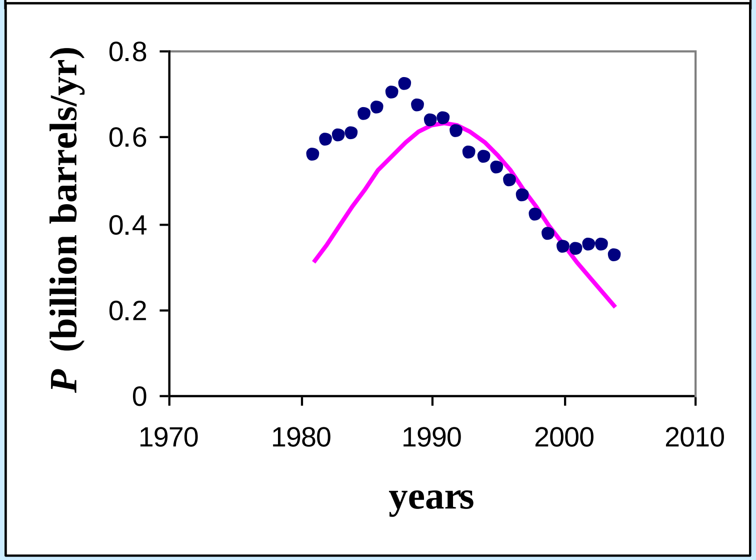


Oil production for North Slope Alaska. Fit with logistic production curve:

r = 0.18 yr $^{-1}$

Q_{tot} = 14 billion bbl

t_m = 1991



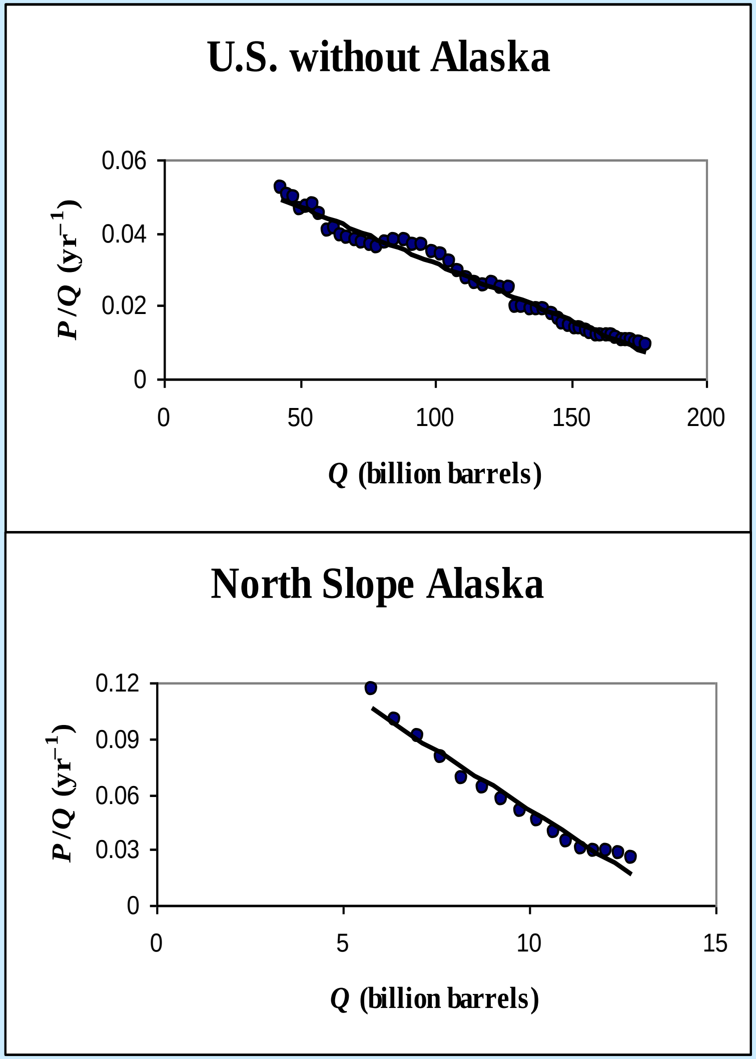
Determining Parameters for Logistic Production Curves

Straight line fits to P/Q vs. Q

y-intercept approximately r

x-intercept approximately Q_{tot}

t_m determined by year in which $Q = Q_{tot}/2$



ANWR Oil Production

Hypothetical logistic production curve with:

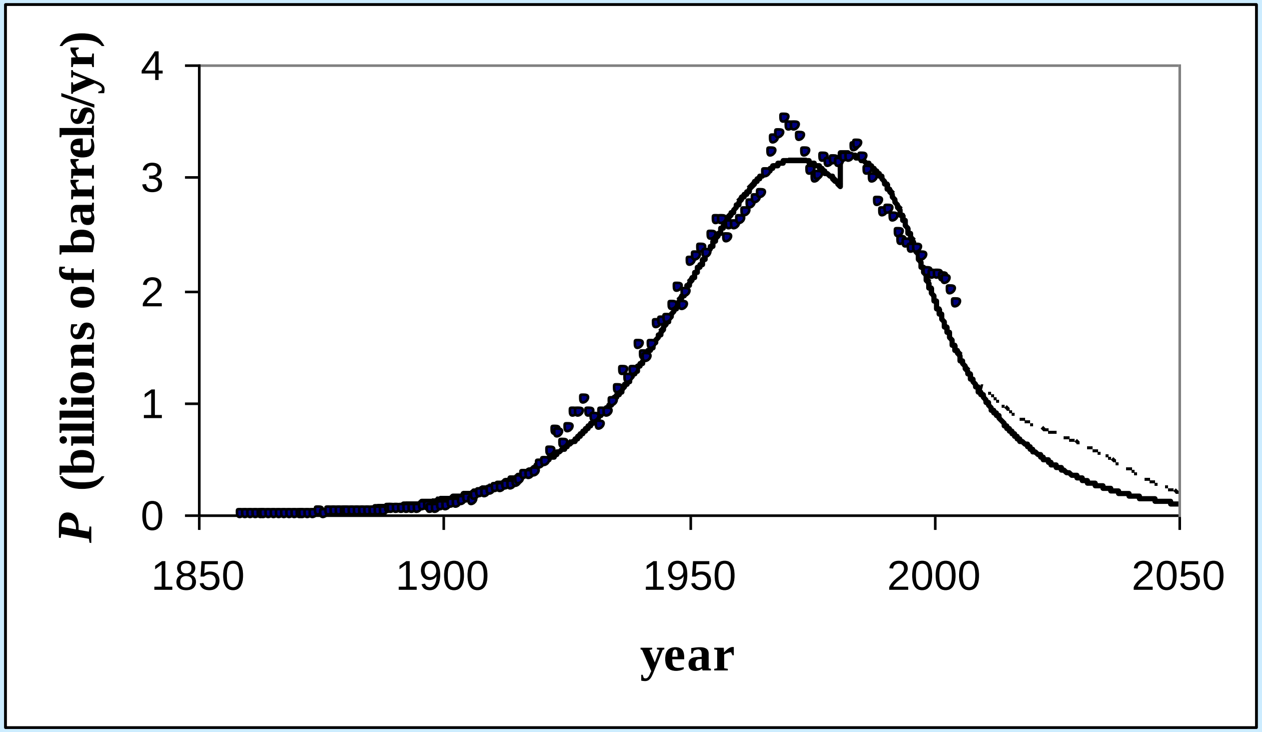
r = 0.12 yr $^{-1}$ (midway between U.S. & North Slope Alaska)

Q_{tot} = 10 billion bbl (USGS mean estimate)

t_m = 2030 (production begins 2010)

Peak production 300 million bbl/yr (USGS mean estimate)

Putting It All Together



U.S. production of oil in billions of barrels per year.
Solid line is a fit to the data projected to 2050.
Dashed line after 2010 is an estimate of total U.S. oil production if oil is extracted from ANWR.

U.S. Dependence on Foreign Oil

“The key point is that recovering oil from ANWR cannot stop the overall downward trend in U.S. oil production. Therefore, recovering this oil is highly unlikely to stop U.S. dependence on foreign oil from growing. At best it will slow the rate of increase of this growing dependence. Indeed, we cannot reasonably expect to end our dependence on foreign oil by increased access to a new supply of U.S. oil. There just isn’t enough oil left in the U.S. The discovery of new oil in the U.S., despite large fluctuations in the data, clearly peaked decades before oil production peaked in 1970. As the 21st century unfolds we will become more and more dependent on foreign oil unless we almost completely eliminate U.S. demand for oil.”

Conclusions

- The logistic growth curve fits real oil production data.
- The toy model captures several features of pressure-driven oil extraction.
- The model provides physical insight into why oil production is bell shaped & declines from the midpoint.
- In the toy model, oil production must peak before the area peaks, which implies increased drilling cannot serve as an indicator of future increased production.
- The U.S. cannot drill itself out of an oil shortage.

Papers

- Abrams D. M. and R. J. Wiener, 2009: A model of peak production in oilfields. *Accepted*, *Am. J. Phys.*

- Wiener, R. J. and D. M. Abrams, 2007: A physical basis for Hubbert's decline-from-the-midpoint empirical model of oil production. *Energy and Sustainability*, ed. C. A. Brebbia and V. Popov, WIT Press.

- Wiener, R. J., 2006: Drilling for oil in the arctic national wildlife refuge. *Forum on Physics & Society of the American Physical Society*, **35**, *No*. 3.